The Nature of Elliptical Galaxies, Proceedings of the Second Stromlo Symposium, Eds. M. Arnaboldi, G.S. Da Costa & P. Saha

Dynamics of Triaxial Stellar Systems

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Abstract: Recent work on the dynamics of triaxial stellar systems is reviewed. The motion of boxlike orbits in realistic triaxial potentials is generically stochastic. The degree to which the stochasticity manifests itself in the dynamics depends on the chaotic mixing timescale, which is a small multiple of the crossing time in triaxial models with steep cusps or massive central singularities. Low-luminosity ellipticals, which have the steepest cusps and the shortest dynamical times, are less likely than bright ellipticals to have strongly triaxial shapes. The observational evidence for triaxiality is reviewed; departures from axisymmetry in early-type galaxies are often found to be associated with evidence of recent interactions or with the presence of a bar.

1. Introduction

Triaxiality began as a plausibility argument (Binney 1978). Since 1975, elliptical galaxies had been known to be rotating too slowly for their shapes to be due to centrifugal flattening. Hence there was no compelling reason for them to be oblate, and triaxiality seemed a natural alternative. The discovery that orbits in triaxial potentials were often regular, i.e. non-chaotic, coupled with the seeming ease with which self-consistent triaxial models could be constructed on the computer (Schwarzschild 1979, 1982) lent further support to the hypothesis. The geometrically simpler alternative – that elliptical galaxies are axisymmetric, and that their slow rotation is due to the cancellation of angular momentum by stars orbiting in opposite directions about the symmetry axis – seemed contrived. Furthermore a growing body of observational evidence suggested that many early-type galaxies were not axisymmetric.

The case for triaxiality is perhaps less compelling now than it was fifteen years ago. Box orbits, the "backbone" of triaxial galaxies, require for their existence a constant-density core where the motion is nearly harmonic. We now know that nature never provides elliptical galaxies with such cores; indeed, massive central singularities may be the norm (Kormendy & Richstsone 1995). It is also clear that nature is capable of making stellar systems that are both pressure-supported and close to axisymmetric (Rubin et al. 1992; Merrifield & Kuijken 1994). The link between velocity anisotropy and triaxiality is thus weakened. Finally, while some early-type galaxies are definitely not axisymmetric, many of these galaxies may be barred S0's or systems that are not fully relaxed – quite different from the original conceptual model of stationary, nested ellipsoids.

In triaxial models that resemble real elliptical galaxies, most of the box-like orbits are stochastic, respecting only the energy integral. The potential of stochastic orbits to induce evolution of the global shapes of elliptical galaxies has long been recognized (Norman et al. 1985; Gerhard & Binney 1985), but recent thinking about this problem has been sharpened by recasting it in terms of "chaotic mixing," the mechanism by which an ensemble of points in stochastic phase space relaxes to a steady state. In a galaxy where the chaotic mixing time is short compared to a Hubble time, nature does not have the freedom to assign arbitrary densities to different parts of stochastic phase space, any more than it can place all of the gas molecules in one corner of a room. The resulting loss of freedom makes it more difficult to arrange stars into self-consistent triaxial equilibria.

2. Cores and Cusps

High resolution observations of elliptical galaxies by a number of groups have consistently shown that nuclear density profiles continue to rise into the smallest observable radius. The non-existence of constant-density cores might have been recognized even before the era of HST. For instance, in M87, a prototypical "core" galaxy, the surface brighntness measurements of Young et al. (1978) imply a deprojected luminosity density that rises as a power-law inside of $\sim 10''$, the nominal core radius (e.g. Richstone & Tremaine 1985, Fig. 1). Galaxies like M87 appear to have cores because of an optical illusion associated with projection onto the plane of the sky. A density profile that varies as $r^{-\gamma}$ at small radii generates a power-law cusp in projection only if $\gamma > 1$. When $\gamma = 1$, the surface brightness exhibits a curving, logarithmically-divergent central profile (e.g. Dehnen 1993, Fig. 1), and for $\gamma < 1$ the central surface brightness is finite. The observed brightness profile of a galaxy like M87, which has $\gamma \approx 0.8$, differs only subtly from that of a galaxy with an isothermal core.

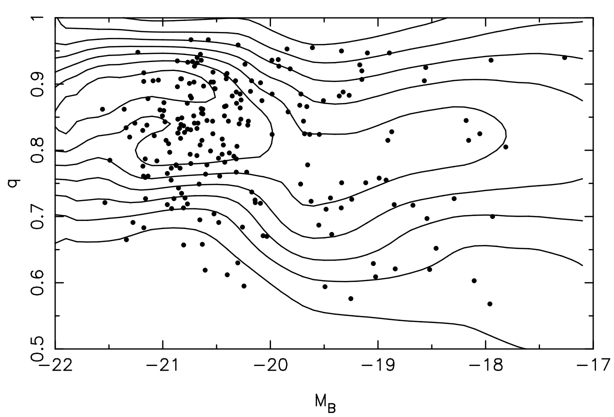

Figure 1. Hubble-type distribution of elliptical galaxies as a function of intrinsic luminosity M_B (Tremblay & Merritt 1996); q is the apparent short-to-long axis ratio. Contours show the frequency function of Hubble types $N(q)$, normalized to unit area at each M_B. There is a striking change in the shape distribution at $M_B \approx -20$.

Thus even if galaxies were distributed uniformly over γ, their surface brightness profiles would appear to fall naturally into one of two categories: the "cores" ($\gamma \leq 1$) and the "power-laws" ($\gamma > 1$). Just such a characterization was adopted based on the first surface brightness measurements from HST (Ferrarese et al. 1994; Lauer et al. 1995). The suggestion of Merritt & Fridman (1995) that *all* elliptical galaxy nuclei might have power-law profiles in the *space* density – differing only in the small-radius slope γ – was beautifully confirmed by Gebhardt et al. (1996), who used a nonparametric algorithm to deproject the HST data from a large sample of ellipticals. Unfortunately, as often happens, the nomenclature has remained frozen and one still hears talk of the "two types" of surface brightness profile (e.g. Lauer, this volume). Here we adopt a more neutral terminology: galaxies with deprojected profiles having $\gamma \leq 1$ are "weak-cusp" galaxies, while values of γ greater than 1 define a "strong cusp."

Even this dichotomy might seem artificial, since it is only through a trick of projection that the value $\gamma = 1$ appears to be special. However there is a more fundamental reason for making the division at $\gamma = 1$. A central density that increases more rapidly than r^{-1} implies a divergent central force. For instance, a spherical galaxy with Dehnen's (1993) density law

$$\rho(r) = \frac{(3-\gamma)M}{4\pi a^3}\left(\frac{r}{a}\right)^{-\gamma}\left(1+\frac{r}{a}\right)^{-(4-\gamma)} \tag{1}$$

has a gravitational force

$$-\frac{\partial\Phi}{\partial r} = -\frac{GM}{a^2}\left(\frac{r}{a}\right)^{1-\gamma}\left(1+\frac{r}{a}\right)^{\gamma-3}. \tag{2}$$

We might expect the character of the "centrophilic" orbits, like the boxes, to change radically as γ is increased past 1. This expectation turns out to be correct, as discussed below. Furthermore there is some indication that the shapes of elliptical galaxies with weak cusps are systematically different than those with strong cusps. Gebhardt et al. (1996) find that $\gamma \gtrsim 1$ for faint elliptical galaxies, $M_V \gtrsim -20$, while $\gamma \lesssim 1$ for brighter ellipticals. As Figure 1 shows, roughly the same absolute magnitude also neatly divides elliptical galaxies into two groups with very different distributions of apparent shapes. This difference may be due in part to the different behavior of boxlike orbits in galaxies with weak and strong cusps.

3. Regular Orbits

The only global integral of the motion in a generic triaxial potential is the energy $E = v^2/2+\Phi$. However we expect extra integrals to exist in the vicinity of stable, periodic orbits. Periodic orbits fill phase space densely, but many of them are unstable, and furthermore the volume of regular phase space associated with a periodic orbit declines rapidly with the order of the resonance. The most important perodic orbits are therefore those associated with the lowest-order stable resonances.

Planar, 1:1 closed orbits – circular orbits in the axisymmetric geometry – exist in a wide variety of triaxial potentials; they disappear only near the centers of triaxial potentials with cores (Merritt & de Zeeuw 1982) where the motion is nearly harmonic and the orbital frequencies incommensurate. The 1:1 orbits circling the long and short axes of a triaxial model are generally stable (Heiligman & Schwarzschild 1979), and perturbations of these orbits produce the long- and short-axis tube families (Kuzmin 1973; de Zeeuw 1985). The fact that tube orbits are present outside the core in almost all triaxial potentials justifies the use of highly simplified models, like the Perfect Ellipsoid, to model the rotational velocity fields far from the centers of real elliptical galaxies (Statler 1991).

The axial orbits constitute a second major class of periodic orbit. The long-axis orbit, when stable, generates box orbits, which are uniquely associated with the triaxial geometry. In models with constant-density cores, the long-axis orbit remains stable from the center out to large radii; it first becomes unstable when the frequency of oscillation along the long axis falls to 1/2 the average oscillation frequency in the direction of the short or intermediate axis. A bifurcation then occurs, with the 2:1 “x-z banana” orbit branching off (Miralda-Escudé & Schwarzschild 1989). In models without constant-density cores, the banana bifurcation can occur at quite small radii, and in models with strong cusps or central singularities, the long-axis orbit is unstable at all energies. As Figure 2 shows, the long-axis orbit in triaxial models with Dehnen’s density law is stable at most energies only when the cusp is weak, $\gamma \lesssim 0.7$, and the figure round, $c/a \gtrsim 0.7$. Both conditions are violated by the majority of low-luminosity ellipticals; thus, box orbits – which require for their existence a stable long-axis orbit – should not be present in most of these galaxies (even assuming that they do not contain nuclear black holes). Brighter ellipticals, which tend to be rounder and to have shallower cusps, may support box orbits, but only at small to intermediate radii where the long-axis orbit is stable, and only if they do not contain nuclear black holes.

Box orbits are strongly populated in the self-consistent triaxial models with large cores (Schwarzschild 1979; Statler 1987). The absence of bona-fide box orbits is not necessarily lethal to the triaxial hypothesis, however, since higher-order resonances can also serve as the generators of boxlike orbits. An obvious candidate is the 2:1 banana orbit that bifurcates from the unstable long-axis orbit. However the range of shapes of the regular orbits associated with the 2:1 resonance is relatively small. The reason is that the banana orbits pass quite near to the origin, and even a modest perturbation is sufficient to drive them into the destabilizing center. This means that the orbits from the 2:1 family are not able to reproduce the shapes of the most highly elongated box orbits. The problem becomes more severe as the elongation of the potential increases, since the bending angle of the banana orbits goes up as the potential becomes flatter (Pfenniger & de Zeeuw 1989).

Schwarzschild (1993) explored the degree to which regular orbits alone could reproduce the mass distribution of triaxial models with scale-free, $\rho \propto r^{-2}$ den-

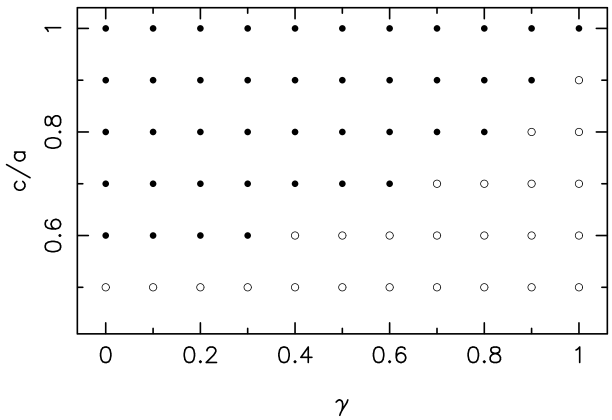

Figure 2. Stability of the long-axis orbit in triaxial models with various short-to-long axis ratios c/a and cusp slopes γ (Fridman 1997). A solid dot indicates stability of the long-axis orbit that just touches the equipotential surface corresponding to the ellipsoidal shell that divides the model into two regions of equal mass. Every model has "maximal triaxiality," i.e. $(a^2-b^2)/(a^2-c^2)=0.5$. For $\gamma>1$, the long-axis orbit is always unstable.

sity laws. He found that self-consistency could not be achieved for highly elongated triaxial models with $c/a = 0.3$. Merritt & Fridman (1996) found that they could not attain self-consistency using just the regular orbits in a model with Dehnen's density law, $\gamma = 2$, $c/a = 0.5$ and $T = (a^2-b^2)/(a^2-c^2) = 0.5$. Work in progress (Merritt 1997) suggests that the restriction to regular orbits in triaxial models with strong cusps limits the degree of triaxiality to values of T less than ~ 0.3 or greater than ~ 0.8. Real galaxies must either avoid these shapes, or else they must incorporate stochastic orbits in order to achieve self-consistency.

4. Chaos and Mixing

Even Schwarzschild's first triaxial model contained a signficant number of stochastic orbits. This was discovered (Merritt 1980) when the orbits were re-integrated using a different computer – about 10% of them generated different occupation numbers than in the original integrations, a result of the well-known sensitivity of stochastic orbits to small perturbations. Orbital stochasticity becomes more important as the central concentration of a triaxial model is increased, since a small core radius or a strong cusp induces instability in orbits that pass near the center. In the scale-free, $\rho \propto r^{-2}$ triaxial potentials investigated by Schwarzschild (1993), many or most of the orbits with boxlike (i.e. stationary) initial conditions were stochastic. Merritt & Valluri (1996) found the same to be true in triaxial potentials with the "imperfect" density law

$$\rho(m) = \frac{\rho_0}{(r_0^2+m^2)(1+m^2)}, \qquad m^2 = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}. \tag{3}$$

For $r_0 = 1$, this reduces to the Perfect Ellipsoid (de Zeeuw & Lynden-Bell 1985), while for $r_0 = 0$ the model has a $\rho \propto m^{-2}$ central density cusp. Even

for $r_0 = 0.1$, the majority of boxlike orbits were found to be stochastic, and the fraction increased as r_0 was reduced to 0.01 or 0.001. Adding a central point mass containing $\sim 0.3\%$ or more of the total mass was also found to be effective at destroying the regularity of most of the box orbits, consistent with the prediction of Gerhard & Binney (1985).

Stochastic motion is qualitatively different from regular motion: a stochastic trajectory is not quasi-periodic, and perturbations of a stochastic orbit grow exponentially with time. Until recently, model builders tended to ignore stochastic orbits, both because they were thought to be rare in triaxial potentials and because their shapes seemed to make them poor building blocks for galaxies. But lately the focus has changed. While model builders may prefer to avoid stochastic orbits, there is no reason for nature to do so; the fraction of stars on stochastic orbits in a real galaxy is probably comparable to the fraction of phase space that is stochastic. Furthermore, there is an interesting new timescale associated with stochasticity, the chaotic mixing timescale (Kandrup & Mahon 1994). An initially localized clump of stars in stochastic phase space will spread as the stellar trajectories diverge; the timescale for the divergence is initially equal to the inverse of the Liapunov exponent that characterizes the orbital instability. Because stochastic motion is essentially random over long periods of time, the probability of finding a single star from the ensemble anywhere in stochastic phase space tends toward a constant; in other words, the distribution of stars evolves toward a steady state. Something similar to this takes place in regular phase space, as the phases of stars on nearby orbits gradually move out of synch. But chaotic mixing is often more efficient than phase mixing, since the region accessible to a stochastic orbit is much larger than the single torus to which a regular orbit is confined, and since the divergence in stochastic phase space grows exponentially with time (Merritt 1996).

If chaotic mixing were always an efficient process, one would expect the stochastic parts of phase space to be fully mixed in real triaxial galaxies. The net effect would be to remove many or most of the boxlike orbits from solution space, and to replace them with the much smaller set of "orbits" (i.e. invariant densities) associated with stochastic phase space at each energy. However the timescale associated with chaotic mixing in triaxial galaxies is a strong function of the degree of central concentration. Goodman & Schwarzschild (1981) found that the stochastic orbits in a triaxial model with a large core behaved essentially like regular orbits for $\sim 10^2$ oscillations. Merritt & Valluri (1996) likewise found a slow rate of chaotic mixing in "imperfect ellipsoids" (Eq. 3) with $r_0 = 0.1$. However when r_0 was reduced to 0.01 or 0.001, the mixing time dropped to a small multiple of the crossing time. Many dynamical systems exhibit such a steep dependence of the chaotic mixing rate on the parameters defining the potential (e.g. Contopoulos et al. 1996).

The strong dependence of the chaotic mixing rate on the degree of central mass concentration in triaxial models leads to an interesting prediction (Merritt & Valluri 1996). Low-luminosity ellipticals, which have strong cusps as well as short dynamical times, ought to be well-mixed; while bright ellipticals, which

have shallower cusps and longer crossing times, need not be. Faint ellipticals are therefore less likely than bright ones to be strongly triaxial. In fact there is a striking change in the apparent shape distribution of elliptical galaxies near $M_B = -20$ (Fig. 1); this is roughly the same magnitude at which the typical cusp slope changes from $\gamma \approx 2$ to $\gamma \approx 1$ (Gebhardt et al. 1996). Tremblay & Merritt (1996) found that the Hubble-type distribution for the faint (strong cusp) ellipticals could be well reproduced under the axisymmetric hypothesis, while that of the brighter ellipticals could not, suggesting that some of the the latter were triaxial. On the other hand, if dynamically signficant black holes are components of most elliptical galaxies – as they seem to be in M32 and M87 – then chaotic mixing of the boxlike trajectories should be efficient regardless of cusp slope. Whichever is the case, mixing will have proceeded farther in the central regions of elliptical galaxies than in the envelopes; thus axisymmetry should be most common at small radii.

5. Do Triaxial Galaxies Exist?

The triaxial hypothesis was strengthened early on by a series of observational studies that detected departures from axisymmetry in many early-type galaxies. Isophote twists were widely interpreted as signatures of triaxiality (Benacchio & Galletta 1980; Leach 1981). But twists at large radii could also be intrinsic, resulting from tidal interactions or accretion in an inclined plane. Many galaxies clearly fall into this category, e.g. Centaurus A. And some galaxies with the strongest twists are probably not ellipticals at all. Fasano & Bonoli (1989) note that significant twisting is only seen in galaxies that deviate from a de Vaucouleurs luminosity profile, and suggest that some fraction of twisted ellipticals are barred S0's. Nieto et al. (1992) note further that many galaxies with large twists exhibit strong changes in the isophote shapes at the radius where the position-angle change is the greatest. They argue that most of these galaxies are barred S0's, and show that both NGC 596 and 1549 – two of the prototypical "twisted ellipticals" – have isophotal morphologies very similar to those of known SB0's.

Kinematical tests for triaxiality have also not fared well. The same trick of projection that causes the isophotes of a triaxial system to twist can also induce a twist in the stellar velocity field (Contopoulos 1956; Binney 1985). Franx et al. (1991) identified an unbiased sample of 38 ellipticals with measured rotation curves along the major and minor axes and tested whether the distribution of kinematic misalignment angles, $\Psi = \tan^{-1}(v_{\rm minor}/v_{\rm major})$, was consistent with various hypotheses about the intrinsic shapes. The found Ψ to be strongly peaked around $\Psi = 0$ (rotation about the apparent minor axis) and $\Psi = \pi/2$ (rotation around the apparent major axis), with most galaxies having $\Psi \approx 0$. A much smaller fraction of galaxies have strong misalignments, $v_{\rm minor} \approx v_{\rm major}$. While this distribution is not inconsistent with triaxiality, it is more naturally reproduced by assuming that $\sim 60\%$ of ellipticals are oblate and $\sim 40\%$ prolate, with perhaps a handful – those with the strongest misalignments – triaxial.

The handful of elliptical galaxies with strong kinematic misalignments are

among the best current candidates for triaxiality. Since they are so few in number, it makes sense to examine several of them in detail here.

NGC 1549 Franx et al. (1989) quote $v_{minor}/v_{major} = 0.87$ for this E2 galaxy. Malin & Carter (1983) note the existence of faint shells and point out that NGC 1549 appears to be interacting with its neighbor NGC 1553. Longo et al. (1994) present rotation curves along two position angles and stress the "very strange kinematical behavior": the minor-axis rotation curve is U-shaped, a likely indicator of a recent interaction. Nieto et al. (1992) argue that NGC 1549 is a misclasssified SB0.

NGC 2749 An E2 galaxy; Jedrzejewski & Schechter (1989) find $v_{minor}/v_{major} \approx 1.1$. But they note that its high major-axis rotation places this galaxy above the "oblate isotropic rotator" line, and argue that it may be a mis-classified S0. They also find strong 5007Å emission in the inner regions suggesting the recent accretion of a gas-rich galaxy.

NGC 4365 and 4406 These E3 galaxies exhibit an extreme sort of kinematic misalignment: the angular momenta of the central and outer regions are nearly orthogonal. Both galaxies are minor-axis rotators at large radii, hence probably prolate, but the rotation near the center is along the major axis. Such strong misalignments suggest that the core material was accreted (Kormendy 1984; Balcells & Quinn 1990), and that different orbital families are populated at large and small radii (Statler 1991).

NGC 4589 Möllenhoff & Bender (1989) find $v_{minor}/v_{major} \approx 0.65$ for this E2 galaxy. Statler (1994a) carried out a detailed analysis of NGC 4589 using the Möllenhoff & Bender velocities and concluded that the galaxy was significantly prolate-triaxial, with $T \approx 0.65$ and $c/a \approx 0.8$. But Möllenhoff & Bender note the "fairly complex" stellar rotation field of NGC 4589, including bumps in the major-axis rotation curve. They argue for the recent accretion of a gas-rich companion in order to explain the prominent minor-axis dust lane.

NGC 5128 Centaurus A is a nearby giant elliptical with a prominent dust lane and an extensive gas disk. Evidence for strong departures from axisymmetry has been adduced from the gas motions (Graham 1979), the stellar velocity field (Wilkinson et al. 1986) and the kinematics of the planetary nebula system (Hui et al. 1995). But this galaxy is almost certainly the product of a recent merger event (e.g. Schweizer 1987); the gas distribution in particular appears to be in a transient state (Bertola et al. 1985).

NGC 7145 An E0 galaxy; Franx et al. (1989) find $v_{minor}/v_{major} = 0.72$. It is a shell galaxy (Malin & Carter 1983) and a member of a close pair. Nieto et al. (1992) argue that this galaxy too is a misclassified SB0.

NGC 1700 This E3 galaxy with a strong cusp has been the subject of probably the most detailed kinematical study to date of a single elliptical. Statler et al. (1996) mapped the stellar velocities out to almost five effective radii along four position angles. The presence of shells (Forbes & Thomson 1992) and strongly box-shaped, almost square isophotes at large radii (Franx et al. 1989) suggest that this galaxy experienced a significant merger or accretion event

within the last several Gyr (Schweizer & Seitzer 1992). The rotational velocity field within $\sim 2.5R_e$ is symmetric, with the exception of a weakly counter-rotating (but aligned) core; the inner isophotes exhibit no twists. Statler et al. found that an oblate model could reproduce these inner data extremely well. Starting at $\sim 3R_e$, both the photometric and kinematic axes begin to twist; but beyond $\sim 4R_e$, the galaxy is clearly not in a dynamically relaxed state, since the velocities reverse along one axis producing a U-shaped rotation curve. Statler et al. argued for a model in which the inner regions of NGC 1700 are relaxed and oblate, while the outer regions are still evolving in response to the accretion of a smaller galaxy. The twists first occur at the radius of transition between the relaxed and evolving regions.

As these examples show, departures from axisymmetry are often accompanied by signatures of recent dynamical interactions and/or by hints that the observed galaxy is a barred S0. While many early-type galaxies are clearly not axisymmetric, it is less clear whether their figures would naturally be described as "triaxial" in the sense of stationary, nested ellipsoids. By contrast, the case for axisymmetry is extremely good in a number of elliptical galaxies, including M32 (Qian et al. 1995) and NGC 3379 (Statler 1994b). And in NGC 1700, which is strongly non-axisymmetric at large radii, the inner regions have apparently chosen to relax to an oblate shape. These axisymmetric galaxies all have strong cusps, and M32 probably contains a massive nuclear black hole as well (Bender et al. 1996). Thus there appears to be some support for the hypothesis that global shapes are correlated with the degree of central mass concentration. But the overall evidence for *persistent* triaxiality in elliptical galaxies remains weak.

There is however another class of stellar system for which triaxiality seems to be increasingly implicated. Kormendy (1982) has emphasized that the bulges of barred spiral galaxies often appear to be misaligned both with the bar and with the external disk, implying that the bulges are not axisymmetric. This "triaxial bulge" or "bar-within-a-bar" phenomenon is now known to be quite common (Wozniak et al. 1995). (With a few notable exceptions – e.g. M31 (Stark 1977) – the bulges of non-barred spirals appear to be accurately spheroidal.) Again, it is not known whether these configurations are transient or long-lived, though N-body simulations suggest that multiply-barred systems can persist for many rotations (Sellwood & Merritt 1994). Nevertheless it is striking that triaxiality – which was first put forward as a way of explaining the *slow* rotation of elliptical galaxies – seems to find its most common expression in rapidly-rotating systems like bulges and bars.

I thank T. Statler and M. Valluri for critical comments on the manuscript.

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